The values $x=0$, $y=1250$ are the smallest values satisfying the conditions when $x=0$. The complete solution is $x=0$, $y=1250a^6$, a being any number.

If $x\neq 0$, $y=2x^2$, and we must have, putting in the values of x and y in terms of ξ and η,

$$(\xi^2-\eta^2)k=4\xi^2\eta^2k^2, \text{ or } \xi^2-\eta^2=4k\xi^2\eta^2,$$

a second relation between ξ and η. But this is evidently impossible, if ξ and η have integral values, as is necessary if x and y are to be integral; for transposing $\xi^2=\eta^2(4k\xi^2+1)>\xi^2$. The only *integral* solutions, then, are $x=0$, $y=1250a^6$.

147. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

If $4n+3$ is prime, $2(1.2.3...4n)+1\equiv 0$ (mod. $4n+3$); and conversely. If $4n+3$ is prime, $(1.2.3...2n)^2-4\equiv 0$ (mod. $4n+3$); and conversely.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

By Wilson's Theorem, $1+(4n+2)!\equiv 0(\text{mod. } 4n+3)$.

$\therefore 1+(4n+2)(4n+1)(4n)!\equiv 0(\text{mod. } 4n+3)$;

$1+[4n(4n+3)+2](4n)!\equiv 0(\text{mod. } 4n+3)$.

$\therefore 1+2(1.2.3.4...4n)\equiv 0(\text{mod. } 4n+3)$.

By a corollary to Wilson's Theorem, $[(2n+1)!]^2-1\equiv 0(\text{mod. } 4n+3)$.

$\therefore (2n+1)^2[(2n)!]^2-1\equiv 0(\text{mod. } 4n+3)$;

$[n(4n+3)+n+1][(2n)!]^2-1\equiv 0(\text{mod. } 4n+3)$.

$\therefore 4(n+1)[(2n)!]^2-4\equiv 0(\text{mod. } 4n+3)$.

$\therefore (4n+3)[(2n)!]^2+[(2n)!]^2-4\equiv 0(\text{mod. } 4n+3)$.

$\therefore (1.2.3.4...2n)^2-4\equiv 0(\text{mod. } 4n+3)$.

The converse follow at once.

AVERAGE AND PROBABILITY.

192. Proposed by PROFESSOR R. D. CARMICHAEL, Anniston, Ala.

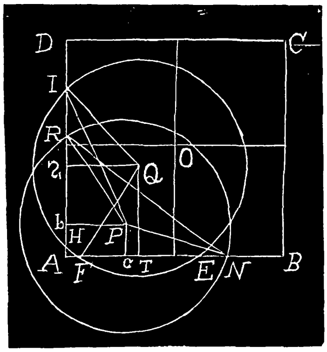
A point is taken at random in a square whose side is $2a$. With this point as center and radius$=a$ a circumference is described. What is the mean area of that part of the circle which lies within the square?

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

By considering the square AO, $\frac{1}{4}$ the original square AC, all possible positions of the circle are taken. Let P, Q be the center of the circle, P taken in the quadrant of a circle radius a, center A, and Q taken in AO

without this quadrant. Coordinates of P, $(Pa, Pb)=(x, y)$; coordinates of Q, $(QT, QS)=(u, v)$.

Then $G=\pi a^2-$area of segment on $IH-$area of segment on FE is the area of Q required.



$K=\frac{1}{2}AR\times AN+$area of segment on RN is the area of P required.

Let $\angle TQF=\theta$, $\angle IQS=\phi$, $\angle NPa=\psi$, $\angle RPb=\rho$.

$\therefore u=a\cos\theta$, $v=a\cos\phi$, $x=a\cos\psi$, $y=a\cos\rho$, $AN=a\cos\rho+a\sin\psi$, $AR=a\cos\psi+a\sin\rho$, $\angle RPN=3\pi/2-\psi-\rho$.

$\therefore G=(\pi-\theta-\phi+\sin\theta\cos\theta+\sin\phi\cos\phi)a^2$;

$K=\frac{1}{2}(3\pi/2-\psi-\rho+\cos\rho\sin\rho+\cos\psi\sin\psi+2\cos\psi\cos\rho)a^2$.

The limits of θ are $\frac{1}{2}\pi-\phi$ and 0; of ϕ, 0 and $\frac{1}{2}\pi$; of ψ, $\frac{1}{2}\pi-\rho$ and $\frac{1}{2}\pi$; of ρ, 0 and $\frac{1}{2}\pi$. Then Δ, the required average area, is

$$\Delta=\frac{\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi-\phi}Ga^2\sin\phi\sin\theta\,d\phi\,d\theta+\int_0^{\frac{1}{2}\pi}\int_{\frac{1}{2}\pi-\rho}^{\frac{1}{2}\pi}Ka^2\sin\rho\sin\psi\,d\rho\,d\psi}{\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi-\phi}a^2\sin\phi\sin\theta\,d\phi\,d\theta+\int_0^{\frac{1}{2}\pi}\int_{\frac{1}{2}\pi-\rho}^{\frac{1}{2}\pi}a^2\sin\rho\sin\psi\,d\rho\,d\psi}$$

$$=\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi-\phi}G\sin\phi\sin\theta\,d\phi\,d\theta+\int_0^{\frac{1}{2}\pi}\int_{\frac{1}{2}\pi-\rho}^{\frac{1}{2}\pi}K\sin\rho\sin\psi\,d\rho\,d\psi$$

$$=\tfrac{1}{6}a^2\int_0^{\frac{1}{2}\pi}[6(\pi-\phi+\sin\phi\cos\phi)\sin\phi-3\sin^3\phi\cos\phi-2\cos^3\phi\sin\phi-2\sin\phi]d\phi$$

$=(\pi-\frac{29}{24})a^2$, ϕ and ρ having the same limits, $\phi=\rho$.

MISCELLANEOUS.

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155 (Incorrectly numbered 151). Proposed by W. J. GREENSTREET, M. A., Stroud, England.

Sum the series $\sum_{r=1}^{r=m}\text{cosec}\left(\frac{2r-1}{4m}\pi+\theta\right)\text{cosec}\,\theta\left(\frac{2r-1}{4m}\pi-\theta\right)$.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

$$S=\sum_{r=1}^{r=m}\text{cosec}\left(\frac{2r-1}{4m}\pi+\theta\right)\text{cosec}\left(\frac{2r-1}{4m}\pi-\theta\right)$$